JIMWLK-KLWMIJ (and beyond) AS THE QCD REGGEON FIELD THEORY

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What are all these letters?

Projectile $|P\rangle$ scatters on a target $\langle T|$.

Target has lots of color charges and large color fields

$$\alpha_T^a(x, x^-)T^a \;= g^2 \, \frac{1}{\partial^2}(x - y) \, \left\{S^\dagger(y; 0, x^-) \; \rho_T^a(y, x^-) \, T^a \; S(y; 0, x^-)\right\}$$

With S - a single gluon scattering matrix (eikonal factor)

$$S(x; 0, x^-) \;=\; \mathcal{P} \, \exp\{i \int_0^{x^-} dy^- \, T^a \, \alpha_T^a(x, y^-)\} \, ; \qquad\qquad S(x) \;\equiv\; S(x; 0, 1) \, .$$

In the eikonal approximation the S-matrix of a projectile scattering on the targte field

$$\Sigma^P[\alpha_T] \;=\; \int d\rho_P \; W^P[\rho_P] \;\; \exp\left\{i \int_0^1 dy^- \int d^2x \, \rho_P^a(x, y^-) \, \alpha_T^a(x, y^-)\right\}$$

The S-matrix has to be averaged over the target wave function (target color field distribution):

$$\mathcal{S}(Y) \;=\; \int D\alpha_T^a \; W_{Y_0}^T[\alpha_T(x, x^-)] \; \Sigma_{Y-Y_0}^P[\alpha_T(x, x^-)] \, .$$

As a function of the total rapidity S evolves as:

$$\frac{d}{d\,Y}\,\mathcal{S} \;=\; -\,\int\; D\alpha_T^a\; W_{Y_0}^T[\alpha_T(x,x^-)]\;\; H\left[\alpha_T,\frac{\delta}{\delta\,\alpha_T}\right]\;\; \Sigma_{Y-Y_0}^P[\alpha_T(x,x^-)]\,.$$

H is a Hermitian "Hamiltonian" which acts either on the left or on the right:

$$\frac{\partial}{\partial Y}\,\Sigma^P \;=\; -\;H\;\,\Sigma^P[\alpha_T]\,; \qquad\qquad \frac{\partial}{\partial Y}\,W^T \;=\; -\;H\;\,W^T[\alpha_T]$$

WE KNOW SEVERAL THINGS ABOUT H

a) H including Pomeron loops must be self dual under the Dense-Dilute Duality transformation:

$$\alpha^a(x, x^-) \to i\frac{\delta}{\delta\rho^a(x, x^-)}, \qquad \frac{\delta}{\delta\alpha^a(x, x^-)} \to -i\rho^a(x, x^-)$$

or

$$S^{ab}(x) \to R^{ab}(x)$$

with "dual Wilson line"

$$R(z)^{ab} = \left[\mathcal{P}\exp\int_0^1 dz^- T^c \frac{\delta}{\delta\rho^c(z,\ z^-)}\right]^{ab}$$

b) In the dense limit (charge density very large - pertinent to the target)

$$H^{JIMWLK} = \int_{x,y,z} K_{x,y,z} \left\{ J_L^a(x) J_L^a(y) + J_R^a(x) J_R^a(y) - 2\, J_L^a(x) S_z^{ab}\, J_R^b(y) \right\}$$

with left and right rotation generators

$$J_L^a[S,x] = tr\left[\frac{\delta}{\delta S_x^\dagger} T^a\, S_x\right], \qquad J_R^b[S,y] = tr\left[S_y\, T^b\, \frac{\delta}{\delta S_y^\dagger}\right]$$

and the Weizsacker-Williams kernel

$$K_{x,y,z} \equiv \frac{\alpha_s}{2\pi^2} \frac{(z-x)_i (z-y)_i}{(z-x)^2 (z-y)^2}$$

c) In the dilute limit (pertinent for the projectile) the Hamiltonian is the DDD dual of H^{JIMWLK}

$$H^{KLWMIJ} = \int_{x,y,z} K_{x,y,z} \left\{ J_L^a[S,x] J_L^a[R,y] + J_R^a[R,x] J_R^a[R,y] - 2\, J_L^a[R,x] R_z^{ab}\, J_R^b[R,y] \right\}$$

H^{KLWMIJ} SUMMS POMERON TREE DIAGRAMMS

This is QCD Reggeon Field Theory?

H generates th energy evolution of the TOTAL CROSS SECTION.

Degree of freedom S^{ab} (or R^{ab}) - scattering matrix of the basic degree of freedom - a single gluon.

That's different form Gribov's RFT obviously. The basic objects are not color singlets. But that's life in perturbative QCD... We have to live with coloured states at least as intermediate ones - like Reggeized gluon.

If this is Reggeon Field Theory, where are the Reggeons?

Let's try to "solve" this RFT.

Well, not really solve it, but come up with an approximation in which we can think about solving it...

S - matrix in RFT

How to calculate S-matrix?

First find eigenfunctions of the H^{KLWMIJ}

$$H^{KLWMIJ}[R, \delta/\delta R]\, G_q[R] \;=\; \omega_q\, G_q[R]$$

Second expand $\Sigma^P[R]$ at Y_0 in G's

$$\Sigma^P[R] \;=\; \sum_q \gamma_q\, G_q[R]$$

Third calculate S

$$\mathcal{S}(Y) \;=\; \sum_q \gamma_q \; e^{-\omega_q\,(Y\,-\,Y_0)} \int DS\; W^T_{Y_0}[S(x)]\; G_q[S(x)]\,.$$

Each factor of 1-R in the "wave function" corresponds to a gluon in the projectile state.

Approximate H^{KLWMIJ} with Hamiltonian which preserves the number of gluons in the incoming state.

Partonic approximation to RFT

Expand H^{KLWMIJ} in $\tilde{R} = R - 1$ - also have to make sure the truncated Hamiltonian is UV and IR finite, like H^{KLWMIJ} itself.

The homogeneous part is

$$H_{part} = \hat{K}_{x,y,z} \Big\{ - 2\, tr\left[\frac{\delta}{\delta \tilde{R}_x^\dagger} T^a (\tilde{R}_x - \tilde{R}_z)\right] tr\left[(\tilde{R}_y - \tilde{R}_z) T^a \frac{\delta}{\delta \tilde{R}_y^\dagger}\right]$$

$$+ tr\left[\frac{\delta}{\delta \tilde{R}_x^\dagger} T^a (\tilde{R}_x - \tilde{R}_z)\right] tr\left[\frac{\delta}{\delta \tilde{R}_y^\dagger} T^a (\tilde{R}_y - \tilde{R}_z)\right]$$

$$+ tr\left[(\tilde{R}_x - \tilde{R}_z) T^a \frac{\delta}{\delta \tilde{R}_x^\dagger}\right] tr\left[(\tilde{R}_y - \tilde{R}_z) T^a \frac{\delta}{\delta \tilde{R}_y^\dagger}\right] \Big\}$$

$$+ 2 \int_{x,z} K_{x,x,z} \left\{ - tr\left[T^a (\tilde{R}_x - \tilde{R}_z) T^a \frac{\delta}{\delta \tilde{R}_x^\dagger}\right] + N\, tr\left[(\tilde{R}_x - \tilde{R}_z) \frac{\delta}{\delta \tilde{R}_x^\dagger}\right] \right\}$$

The old curiosity shop: The Spectrum

Vacuum - no Gluons, completely "white" state

$$\frac{\delta}{\delta \tilde{R}} |0\rangle \;=\; 0$$

One gluon states - Reggeons in color representation λ:

$$G_q^\lambda \;=\; \int_x e^{iqx} \eta_{cd}^\lambda \, \tilde{R}^{cd}(x) \, |0\rangle$$

with λ=1, 8_A, 8_S, 10, $\overline{10}$, 27, and $\mathcal{R}_7$

Eigenvalues:

$$\omega_q(\lambda) \;=\; \frac{\bar{\alpha}_s}{\pi} \frac{C(\lambda)}{2N} \int_\mu d^2k \, \frac{q^2}{k^2\,(q-k)^2} \;=\; \bar{\alpha}_s \, \frac{C(\lambda)}{2N} \ln \frac{q^2}{\mu^2}$$

λ - is the color representation exchanged in the t - channel

Two gluons states - "Pomerons" and "Odderons"

We find "bound states" of the Reggeons.

All color singlets.

Discrete symmetries: Charge conjugation, Signature $(R \to R^\dagger)$, Parity,

$$G_\lambda^{C\,(SP)} = P_{ab}^{\lambda\,cd} \int_{u,v} \tilde{R}^{ab}(u)\,\tilde{R}^{cd}(v)\,\Psi(u,v)$$

where $P_{ab}^{i\,cd}$ projectors from $\lambda\lambda$ to the singlet.

$\Psi(u,v)$ are BFKL eigenfunctions.

The zoo of "Pomerons" (C-even states) - bound states of two identical λ "Reggeons".

Their trajectories

$$\omega_\lambda = \frac{C(\lambda)}{N}\omega_{BFKL}, \qquad \omega_{27} = 2(1+\frac{1}{N})\omega_{BFKL}$$

$$\boxed{\text{Odderons - C-odd states.}}$$

S,P -odd $(8_A, 8_S$ bound state) - BLV Odderon.

But also a S-odd, P - even bound state of 10 and $\overline{10}$ that grows with energy, with

$$\omega = \omega_{BFKL}$$

Curiouser and curiouser. But where have they been before?

- VERY IMPORTANT!

All the solutions except the ones that appear in BFKL approach (Reggeized gluon and BFKL Pomeron) require each gluon of the projectile to multiply scatter on the target field.

If we disallow multiple scattering, that is take $\tilde{R}^{ab} = T^c_{ab}\frac{\delta}{\rho^c}$ in the partonic Hamiltonian, only BKP states remain in the spectrum for any gluon number.

So the coupling of all our new friends is suppressed by powers of α_s or $1/N$.

It's different physics, but still, they are all there. Are they important? Possibly no less importnat than higher BKP states - indication of what unitarizes the cross section beyond BFKL.

What do we know of RFT beyond JIMWLK-KLWMIJ?

- Full H^{RFT} is Hermitian with positive spectrum - the evolution is unitary.

In fact even JIMWLK

$$H^{JIMWLK} = \int_z Q_a^i(z) Q_a^i(z)$$

with

$$Q_a^i(z) = \int d^2x \frac{(x-z)_i}{(x-z)^2}[S^{ab}(z) - S^{ab}(x)] J_R^b(x)$$

one gluon production amplitude.

So the unitarity violation of BFKL is the result of truncation of H^{JIMWLK}.

- Full H^{RFT} is DDD invariant.

The DDD transforms the "white" state (Yang) into the "black" state (Yin).

$$|Yang\rangle = \delta[\rho]; \qquad S|Yang\rangle = |Yang\rangle; \qquad \frac{\delta}{\delta R}|Yang\rangle = 0$$

$$|Yin\rangle = 1; \qquad \langle Yin|S(x)...S(y)|Yin\rangle = 0; \qquad \frac{\delta}{\delta S}|Yin\rangle = 0$$

$$H^{RFT}|Yin\rangle = H^{RFT}|Yang\rangle = 0$$

Also "accidentally" true for H^{JIMWLK} and H^{KLWMIJ}!

$$\boxed{\text{Vacuum is doubly degenerate: DDD symmetry is spontaneously broken.}}$$

- There are two degenerate towers of states.

"GLUONS" - live above Yang

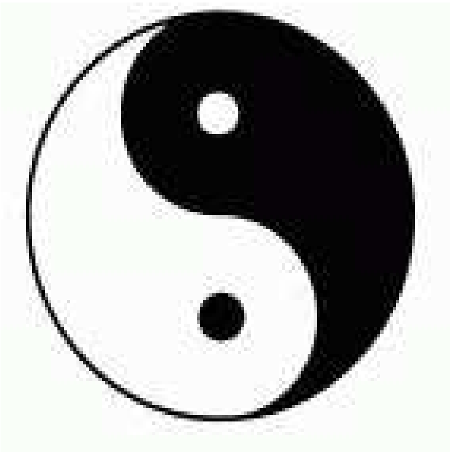
$$G_n = R(x_1)...R(x_n)|Yang\rangle$$

S - matrix is unity at all points in the transverse plane except $x_1, ..., x_n$

"HOLES" - live above Yin

$$H_n = S(x_1)...S(x_n)|Yin\rangle$$

S - matrix vanishes at all points in the transverse plane except $x_1, ..., x_n$
(H^{part} is self dual - both towers are in the spectrum.)



Do we care?

Both towers contribute to the calculation of S-matrix.

Expand W of projectile and target in the basis of eigenstates of H:

$$W^{P(T)}[S] \,=\, v^{n/2} \sum_{i:\,\omega_i} \gamma_i^{P(T)}\, G_i[S] \,+\, v^{-n/2} \sum_{i:\,\omega_i} \eta_i^{P(T)}\, H_i[S]$$

Here v - is "volume"

The S - matrix becomes

$$\mathcal{S}(Y) \,=\, 1 - \eta_0^P\, \eta_0^T + \sum_{i:\,\omega_i > 0} e^{-\,\omega_i\,Y} \left[\gamma_i^P\, \eta_i^T \,+\, \eta_i^P\, \gamma_i^T \,+\, \gamma_i^P\, \gamma_i^T\, v^n \int D\rho\, G_i\, H_i \right] .$$

$\boxed{\text{The last term is the contribution of Pomeron loops.}}$

Conclusions?

Not after 15 minutes talk.

But many open questions.